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New Proof of Einstein's Clock Paradox by General Relativity

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ABSTRACT

A proof is given based on Einstein's general relativity to explain an old problem, namely, the clock paradox of Einstein, which originated in his 1905 paper on special relativity. Essentially the problem involves time intervals which are said to dilate by the Lorentz factor. Recall that this factor applies only in inertial systems to relative motion at velocities constant in both magnitude and direction. Einstein, however, applied it to a clock situated on the earth's surface which obviously is not at constant velocity, since its direction is changing continually. The clock is therefore in a noninertial setting, yet tests show that this factor accounts for the time dilation perfectly for all cases of circular motion. In the paper it is explained using Einstein's Equivalence Principle and general relativity, that time dilation in motion will occur only when the velocity is continually changing either in magnitude or in direction. The clock paradox and the related paradox of the twins are therefore true but only in accelerating systems.

Keywords: Einstein's clock paradox, equivalence principle, general relativity.

INTRODUCTION

We shall revisit a celebrated paradox which has become a much-discussed staple of special relativity, and shall show that its proper explanation can only be given by Einstein's general relativity. The paradox has its origin in his 1905 paper on special relativity where he said that a clock situated on the earth's surface, away from the poles, would slow down by the factor $1/\gamma$, where γ is the Lorentz factor given by $\gamma=1/\sqrt{1-v^2/c^2}$;

here v is the speed at the location where the clock is stationed and c is the speed of light. Here and in the following we use standard notations without further explanation; we shall use the word speed to denote the magnitude of the velocity, the latter being of course a vector. Recall that the Lorentz factor was derived for inertial systems assuming a constant velocity, that is, constant both in magnitude and direction; it is often forgotten that the Lorentz factor holds only for unaccelerated uniform rectilinear motion. On the earth the clock's speed is constant, but not its velocity since its direction changes continually. As such, the clock is in a noninertial setting and subject to a centrifugal force, in this case very small. The analysis of retardation of time-dependent processes in noninertial systems by the use of the Lorentz factor and special relativity is therefore incorrect, but it became the standard practice when it was not known how to treat the problem by other means suitably.

Quite a few textbooks do attempt to explain the paradox for a clock travelling at constant velocity but an examination of their proofs reveals that the Lorentz factor γ is embedded in the beginning itself in one of the dynamical quantities, for example, in the relativistic mass or energy; then γ travels well camouflaged with the flow of the argument, only to emerge finally in the equation $dt^{'}=\gamma dt$, where $dt^{'}$ is recognised as the dilated value of dt. This is an example of circulus in definiendo where something which is to be derived is itself assumed initially as a fact.

We shall now demonstrate a satisfactory treatment based entirely on Einstein's Principle of Equivalence and general relativity, and show that a physical slowdown of time-dependent processes occurring in matter in motion, whether it is a particle or a clock, can occur only in a noninertial setting where the velocity is continually changing in magnitude or in direction (or in both). It will follow that a constant velocity will have no intrinsic effect on the evolution of time-dependent processes, in stark contrast to what is stated on this topic in the standard textbooks on special relativity. This is supported by the fact that all the observed cases of time retardation have occurred only in noninertial systems; this fact has served as the motivation to reexamine the time retardation phenomenon in this paper.

THEORY

The Equivalence Principle establishes a one-to-one correspondence between a force due to a gravitational field and an inertial force due to acceleration. We know that gravitation causes inertial acceleration, so conversely inertial acceleration must cause a gravitational field localized to the body that is undergoing acceleration. In the converse case, the acceleration can be either positive or negative. Using standard notations, Schwarzschild's metric is given by

$$ds^2=g_{11}dx^2+g_{22}dy^2+ g_{33}dz^2+g_{44}dt^2. \quad (1)$$

Since the particle's position does not change in its own reference frame, but of course time must advance, only the last term in (1) has relevance now; hence $ds^2=g_{44}dt^2$. For a gravitating body treated as a point-mass M

situated at r=0, Schwarzschild showed that $g_{44}=c^2(1+2\Omega/c^2)$, where $\Omega=-GM/r$ is the potential of the field at radius r; then

$$ds^2=c^2(1+2\Omega/c^2)dt^2. \quad (2)$$

Let Δt represent the time duration of some repetitive process, for example, the period of vibration. Since the line interval Δs is independent of its location in 4-space, then for a given Δs if changes in Ω occur, it follows from (2) that Δt can have only such values as will keep the value of $\sqrt{(1+2\Omega^2/c^2)}\Delta t$ constant. By applying this criterion to a particle which is under the influence of a localized field generated by its own changing velocity, the time dilation factor for any given case can be easily determined, if Ω has a value which is appropriate for that particular case. This criterion shall apply whatever be the reason for the changes in Ω. Although Schwarzschild's metric was derived for the radially symmetric field of a point-mass, we can safely apply it to the localized self-generated field within any particle undergoing a velocity change, treating it as an infinitesimal sphere, such as an atom; this atom could exist by itself or be any of the atoms constituting an object. All such atoms are affected in an identical manner, so the object can be matter of any arbitrary shape, density or size. We shall now analyze three well-known cases where a slowdown of clock rates was observed.

(i) Circular motion: Let a particle be in circular motion at radius r and constant angular velocity ω. Its angular acceleration will be $\omega^2 r$, and the force field-strength is given by $-\partial\Omega/\partial r$. Equating these quantities we obtain $\Omega=-\omega^2 r^2/2=-v^2/2$, where $v=\omega r$ is the particle's speed. Using this value of Ω and denoting the time interval when the velocity is v by Δt and that when at rest (v=0) by Δt_o, the criterion that $\sqrt{1+2\Omega/c^2}\Delta t$ shall remain constant for any value that Ω may assume, immediately yields $\Delta t_o=\sqrt{1-v^2/c^2}\Delta t$. Hence

$$\Delta t=[1/\sqrt{1-v^2/c^2}]\Delta t_o. \quad (3)$$

Thus the time interval at velocity v is dilated by the factor $1/\sqrt{1-v^2/c^2}$. If the particle is radioactive and undergoing decay, its time rate of decay will slow down by the factor $\sqrt{1-v^2/c^2}$.

(ii) Rectilinear motion at constant acceleration: If a is the acceleration and u, v the initial and final speeds of a particle in rectilinear motion, the change in the potential of the particle as it moves through a distance s, will be given by $\Omega=-as=-(v^2-u^2)/2$. Using this value of Ω, and denoting the time intervals pertaining to u, v by Δt_o, Δt respectively, and also noting that Ω is zero initially at speed u, we obtain

$$\Delta t=[1/\sqrt{1-(v^2-u^2)/c^2}]\Delta t_o. \quad (4a)$$

If the particle were to start from rest (u=0), then dt_o would dilate to the value

$$\Delta t=[1/\sqrt{1-v^2/c^2}]\Delta t_o. \quad (4b)$$

(iii) Rectilinear motion at constant deceleration: The acceleration being negative now, we simply replace a by −a in the above equations and get

$$\Delta t=[1/\sqrt{1-(u^2-v^2)/c^2}]\Delta t_o. \quad (u>v) \qquad (5a)$$

If the final speed v is zero, then

$$\Delta t=[1/\sqrt{1-u^2/c^2}]\Delta t_o. \qquad (5b)$$

This formula has been derived in the literature by combining the Lorentz transformation of special relativity with the Equivalence Principle of general relativity, but the derivation is burdened by unrealistic assumptions. The analysis presented above is straightforward and shows that if the velocity remains constant in rectilinear motion (u=v), no physical dilation of time intervals can occur. It follows that Einstein's clock paradox is true provided the clock's velocity is continually changing.

DISCUSSION

The dilation factor in (3) accounts for the slowdown of radioactive decay of particles moving at high speeds in circular particle accelerators, with accelerations of the order of 10^{16} m s^{-2} or more. It also accounts for the slowdown of atomic clocks that were flown in airplanes around the world by Hafele and Keating[1]. These flights of course constituted circular motion, which atomic clocks having nanosecond precision can easily sense. In the experiment of Vessot et al.[2], the dilation factor in (4b) was instrumental in accounting for the drop in the frequency of signals emitted by a maser that was located in a spacecraft which was allowed to fall back to the earth freely in the earth's gravity (after having been lifted vertically to a great height by a rocket).

The dilation factor in (5b) explains why the μ-mesons created by cosmic rays entering the earth's atmosphere have considerably longer lives before they decay, when they are plunging through the atmosphere. This is because these mesons have initial velocities (u) close to that of light, thus making the denominator in (5b) very small in value. The mesons, however, decelerate to low speeds very rapidly due to collisions with atmospheric gas molecules, at rates as high as 2×10^{14} m s^{-2} as measured by Frisch and Smith[3]. The assumption made by many authors that atmospheric mesons travel with constant, or even nearly constant, velocity was shown to be false by Kantor[4]; it was, however, a convenient one to make! By way of interest, the first physicist to mention the fact that the dilation of the lifetimes of atmospheric μ-mesons was a relativistic phenomenon was Homi Bhabha[5].

The particle accelerator experiment has been repeated numerous times by various groups. These experiments as well as the experiment of Vessot et al. were performed with high precision, and the results obtained agreed with the predicted values almost perfectly. In the experiment of Frisch and Smith involving atmospheric mesons, and that of Hafele and Keating, the conditions for conducting them were far from ideal for practical reasons, so the agreement was only fair; this was to be expected but it was definitely supportive of the theory. Note that atmospheric mesons can be studied only statistically as a group,

and that is where errors of judgement creep in. Further, we have assumed their deceleration rate to remain constant, but in reality it is very nonlinear due to the nonlinear variation of atmospheric density.

In spite of the velocity changes that obviously existed in these experiments, the above authors all assumed a constant velocity in order to justify their use of the Lorentz factor and special relativity. This followed the trend set by the authors of all the textbooks on special relativity. The reason for this is explained in the last but one paragraph below. If the kinematic dilation of time were a physical effect, then the concomitant kinematic length contraction should also be a physical effect and should occur only in the direction of travel, but such a thing has never been observed.

There is, however, one case (see[6]) where the time interval dilated physically and the velocity was constant. It involved tracks of high-velocity mesons in a bubble chamber, but here the track lengths also got dilated by the factor γ. The reason for the dilations of both the track length and the time of travel of a meson before it decayed, was the relativistic increase in the meson's mass, and hence momentum, again by the factor γ. The greater relativistic momentum therefore pushed the meson to a longer distance than the distance the Newtonian momentum is capable of pushing; hence the duration of the meson's travel also increased accordingly. The dilations of the track length and of the travel time are both consequences of the increase in the relativistic mass. It follows that dilation of time was not the cause of the increase in track length. It will be noted that in dynamical situations of the kind just analyzed, time intervals and lengths both dilate physically at high velocities, while in kinematical situations dilation of time intervals is always accompanied by contraction of lengths (Lorentz contraction). Kinematic time and length changes are not real in the physical sense (see[4]), and are reversible once the velocity is reduced to zero. On the other hand, the physical slowdown of a clock due to velocity changes is never reversible; the time "lost" is lost forever.

It is remarkable that the time dilation factors in (3), (4b), (5b) for the different cases all have exactly the same generic form: $1/\sqrt{1-v^2/c^2}$, and equally remarkable is the resemblance of this expression to the Lorentz factor. This is mere coincidence but it explains why the physical slowdown of clocks was explained in the past by invoking the Lorentz factor of special relativity, especially in the case of atmospheric mesons for which (5b) "works" even if the initial velocity is assumed to remain unchanged. We have shown that physical time slowdowns can be fully explained by Einstein's Equivalence Principle and general relativity, and that such slowdowns cannot occur at constant velocities.

Finally we remark that the physical time slowdown we have considered results in reducing the frequency of all vibratory phenomena within the accelerating particle itself. This has often been called the Doppler effect due to acceleration, as in[2]. Although acceleration is the root cause, this is not an appropriate choice of words because the classical Doppler effect of special relativity and acoustics causes frequency changes not at the source but at the receiving end, and is purely velocity-dependent. As is well-known

both acceleration-dependent as well as velocity-dependent effects on the frequency have to be considered when analyzing experimental data, since general and special relativity play their separate roles simultaneously.

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